Scand. J. of Economics 101(2), 163–172, 1999

Amartya K. Sen's Contributions to the Study of Social Welfare

Kenneth J. Arrow
Stanford University, Stanford, CA 94305-6072, USA

I. Introduction

The oeuvre of Amartya K. Sen is very considerable in magnitude, nearly twenty books (with two forthcoming) and about 250 articles, mostly in learned journals but also in the higher level of the popular press, and lectures. It has nevertheless a very considerable degree of unity. It is motivated throughout by concern for the welfare of the individuals in an economy, with special interest in the lower part of the income spectrum; starvation, poverty, and economic development have been the objects of fruitful, important, and socially valuable study and have motivated his basic research into the concepts of individual and social welfare.

All of this work is of very high quality and makes important contributions to our understanding and to the formation of policy. But I will isolate for special consideration that large subset which revolves about the conceptual question: what is meant by, "better off socially"? A major portion of his work falls directly into this category. The value of Sen's analysis of social welfare and the conceptual questions relevant to it have now been recognized by the award of the Nobel Memorial Prize in Economic Science.

Unlike some other work which has been awarded this Prize, especially in economic theory, we cannot do justice to Sen's work in social welfare on the basis of one or two seminal papers, although there have been several such, as I will point out. Rather it is the work as a whole and the way the various parts interplay that must be understood to see the importance of Sen's contribution. His exploration of the notions of social welfare takes place at every level of analysis, formal-mathematical, conceptual, and empirical. It is by far the most comprehensive study of its kind, drawing on profound understanding of both economics and moral philosophy. It is useful to note that he is regarded by philosophers as one of their leading lights, and he has published extensively in philosophical journals.

In view of the comprehensive and far-flung nature of his work, it is not surprising that his role has been somewhat different in various specific aspects. In some, indeed, he has opened and defined new fields with strong results. In others, he has perfected and enriched earlier results or contributed along with others. In still others, he has raised new inquiries whose chief value has been the stimulation to others. What is striking is the consistency

of his thought, so that even the most formal work is motivated by philosophical and ethical considerations, while his most interpretive and broadly ranging papers remain compatible with rigorous technical analysis.

Because of the wide range of his work on social welfare, I will discuss it in more detail under five headings: (1) formal theories of social choice; (2) the formal theory of individual preference and choice; (3) the conceptual meanings of choice, welfare, and utility; (4) the measurement of social welfare as reflected in inequality, poverty, and real social income; and (5) empirical analyses of famine and nutrition related to Sen's work on poverty and its moral implications. In Section VII, I briefly review a sample of his work in fields other than the analysis of social welfare.

One general characteristic of Sen's work must be stressed. He is especially concerned with the distribution of welfare however that concept is understood. Averages for him conceal information relevant to the formulation of sound ethical judgments. He has therefore emphasized the measurement and meaning of inequality and in particular poverty as a morally and economically special category.

II. Formal Theory of Social Choice

A major part of Sen's work has dealt with the formal theory of social choice. The basic paradigm in this field has been that of Arrow (1951).[1] It is in the tradition of welfare economics, which has sought to provide a rational justification for choice among alternative possible economic policies based on the preferences of individual members of society. Assume that social choice, like individual choice, is expressed by a preference ordering over (social) alternatives, so that the alternative chosen from any given feasible set of alternatives is the most preferred one. Hence, one formulation of social choice is that of defining a social ordering governing social choice for each *profile* of orderings, one for each individual. In other words, social choice is defined by a mapping from profiles to social orderings. One can then state some desirable properties of this mapping and ask if any such mapping (*social welfare function* in Arrow's and Sen's terminology) exists; if social welfare functions do exist, one can attempt to characterize them in some useful way.

Arrow's conditions can be stated roughly as follows: (U) the social welfare function is defined for all profiles; (P) if the profile is such that everyone prefers alternative x to alternative y, then the social ordering sets x above y (the Pareto principle); (I) the choice between two alternatives depends only on the individual preferences between those two alternatives

[1] Dates in parentheses refer to references listed at the end of this article.

(independence of irrelevant alternatives); and (*ND*) the social ordering does not always agree with the preferences of any single individual (non-dictatorship). Arrow then showed that there is no social welfare function satisfying all of these conditions.

This result has given rise to a large literature, to which Sen has been the outstanding contributor; the best survey is Sen's book *Collective Choice and Social Welfare* [1970].[2] There are several directions to go for changing the negative result to a positive one; they all obviously require some change in the conditions imposed. One is to enrich the informational base, so that the social ordering depends not only on individual orderings but also on cardinal values and, more significantly, on some kind of interpersonal comparisons. A second is to restrict the range of profiles for which the social welfare function is defined, on the basis of some *a priori* assumptions as to the possible kinds of orderings individuals may have. Indeed, even before Arrow's Theorem, Black (1948) had in effect shown that making pairwise choices by majority voting determines a social ordering (obviously satisfying (*P*), (*I*), and (*ND*)) if the social alternatives can be thought of as arrayed along one dimension and if each individual's preference ordering over this array is single-peaked.

Sen has developed both of these research directions, but, more importantly, he has introduced a new statement of the problem which brilliantly combines simplicity and depth in his discussion of the impossibility of a Paretian liberal [I, 1970a]. As will be discussed in Section IV, Sen has objected to the view that social choice should be completely determined by individual welfare comparisons. In particular, this broader perspective has drawn him into studying the conception of rights. Each individual is taken to have the right to make certain choices among social states. To take the least controversial examples, the choice between two social states that differ only in what one individual reads or in the color of the paint in the interior of an individual's house should be made by that individual. Sen then showed very simply that, even without requiring independence (*I*), the conditions (*U*), (*P*) and the "minimal liberalism" requirement, that for each individual there should be at least two social states between which that individual's choice is decisive, lead to a contradiction. This is very surprising; both the Pareto judgment and the idea that each individual has some private domain of choice, even if others would make different choices over that domain, are hard to deny; and independence, which on the whole is central to most variations of the Impossibility Theorem, is not assumed here. The paradox arises because "nosy" preferences of others about choices that are in an

[2]Dates in brackets refer to publications by Sen listed in the Bibliography. Those preceded by a Roman numeral refer to the subject headings of articles written by Sen; those without a Roman numeral refer to his books.

individual's domain of private choice enter into the Pareto judgment. The result is not only surprising analytically but also addresses profound ethical questions on the relation between even the vestigial remnant of utilitarianism contained in the Pareto principle and the existence of individual "rights," a scope (however small) over which the individual has complete control. Sen's work has sparked both a technical literature, e.g., Gibbard (1974) and Suzumura (1978), and contributed to philosophical emphasis on rights from very diverse critical viewpoints, e.g., Nozick (1974) and Dworkin (1978).

A very different line of analysis pursued by Sen has concerned the possibility of resolving the social choice problem by assuming that the range of individual preferences which need to be aggregated is narrowed from the universal range postulated in condition (*U*). In particular, he has stated conditions on profiles under which pairwise majority voting leads to an ordering. Inada (1964) had found two conditions other than Black's single-peakedness of preferences, and Ward (1965) had generalized Black's condition. Sen [I, 1966] introduced a condition (extremal value restriction) which included all previous ones: in any triple of alternatives, there are one alternative and one value (best, middle, or worst) such that no one ranks that alternative at that value. Later, Sen and P. Pattanaik [I, 1969) found necessary and sufficient conditions whereby majority voting yields a well-defined first choice. Sen thus both made a major early contribution to this particular subfield and collaborated in establishing the definitive results.

More representative of Sen's general position is his systematization of the information constraints on social choice. As he observes in his article "Interpersonal Aggregation and Partial Comparability" [I, 1970b], Arrow's assumption of interpersonally incomparable ordinalism is an extreme case. Interpersonally incomparable cardinalism does not get us any further in avoiding impossibility results [1970, Theorem 8*2], a result greatly deepened by Kalai and Schmeidler (1977). In effect, the assumption that individual judgments, ordinal or cardinal, cannot be compared implies that social judgments must be invariant under a wide range of independent transformations of individual preferences (all monotone transformations in the ordinal case, all affine transformations in the cardinal case). For any degree of interpersonal comparability (e.g., that the ratio of unit utility changes for two individuals is judged to lie between two prescribed limits), the invariance requirements on the social welfare function are correspondingly lightened, and the possibility of finding an acceptable social choice procedure increased. For a survey of this point of view in subsequent research, see Sen [I, 1977b].

The research stemming from Sen's argument has indeed been rich. This is one of the areas where Sen's contribution has been the formulation of the question and the inspiration of others. It is, of course, well known that complete interpersonal comparability of unit utility differences permits –

and makes meaningful - the sum-of-utilities criterion as satisfying conditions (*P*), (*ND*), and (*I*) (the last suitably reinterpreted to admit the additional information). Following Sen's stimulus, Hammond (1976) and Strasnick (1976) formulated a different informational basis. They admitted interpersonal *ordinal* comparisons, e.g., that individual i in state x is better off than individual j in state y. This assumption implies that social welfare judgments be invariant under the *same* monotone transformation of all individual preferences.

Hammond and Strasnick independently showed that under this informational constraint, the "leximin" and "leximax" principles satisfy all the social choice conditions above. They are in fact the only possible principles satisfying these conditions and some mild additional ones. By "leximin" is meant the following: for each of the social alternatives, x and y, rank the individuals in increasing order of utility (this operation is meaningful under the informational constraint). If the worst-off under x is better off than the worst-off under y, choose x (or vice versa). If the two worst-off individuals are equally well off, then compare the two second-worst-off individuals and so forth.

The leximin principle is reminiscent of Rawls's "difference principle" (1971), but it must be acknowledged that Rawls gives it a different interpretation. To Rawls, the comparison is in terms not of well-being but of "primary goods," those that give rise to the possibility of well-being. Sen has developed this theme further (see Section IV below).

Sen has contributed to the technical development, as well as setting the terms of the argument. He notes that the leximin principle is not very persuasive by itself for a large society. It is, however, very reasonable if there are only two individuals. But, in conjunction with conditions (*U*) and (*I*), leximin for two-member societies entails leximin for any society; see Sen [I, 1977b, Section 6].

A last example of Sen's seminal role in the area of formal theories of social choice is his formulation of the "Weak Equity Axiom" in his lectures [1973]: an individual getting less utility out of any given income than another should get a higher income (incentive effects aside). That is, disabilities should be compensated for. This axiom is in general inconsistent with utilitarianism and has revived interest in equality as a criterion independent of diminishing marginal utility.

III. Formal Theories of Individual Choice and Rationality

Reflection on social choice provided a new stimulus for studying individual choice and the meaning of rationality. Gossen, Jevons, Walras, Menger, and Edgeworth had introduced the hypothesis that consumer choice of commodity bundles was determined by maximization of a utility function subject to a

budget constraint. Irving Fisher and Vilfredo Pareto observed that the cardinal nature of the utility function was irrelevant; only an ordering of commodity bundles need be postulated. Samuelson (1938) introduced a change in orientation in his concept of revealed preference. The choices made, not an underlying ordering or utility function, were primary. Conditions of rationality were imposed on the choices. As shown later by Ville (1946) and Houthakker (1950), suitably strong conditions on the demand functions implied the existence of an underlying ordering which rationalized choice.

Arrow (1959) observed that if choice from finite sets (rather than budget sets) was assumed to be defined, the equivalence between choice functions defined by maximizing a given ordering and choices satisfying certain consistency conditions took a somewhat different form. This line of analysis invites taking apart the conditions on the choice function (the function defining the choices or choices made from any given opportunity set) and showing the equivalence of each to some form of rationality weaker than an ordering. This study has been pursued by several writers, in particular Uzawa (1956), Herzberger (1973), and Sen. Sen's article, "Choice Functions and Revealed Preferences" [IV, 1971], is the epitome of this work, summing up and extending all previous studies. There are many results; the following is a typical example. Start with a choice function, $C(S)$, mapping each finite opportunity set S into the subset of chosen elements. Define, "revealed preference," R, to mean:

(xRy) if and only if, for some S, x belongs to $C(S)$ and y belongs to S.

State the Weak Congruence Axiom (WCA):

If xRy, then for any S such that y belongs to $C(S)$ and x belongs to S, x must also belong to $C(S)$.

Also state properties (α) and (β):

(α): If S is a subset of T and x belongs to both S and $C(T)$, then x belongs to $C(S)$.
(β): If S is a subset of T and x and y any two elements of $C(S)$, then either both belong to $C(T)$ or neither does.

Then the following theorem holds:

Theorem. *A choice function satisfies WCA if and only if it satisfies both property (α) and property (β).*

Note: Sen also shows that WCA is equivalent to the statements that R is an ordering and that $C^*(S) = C(S)$, where $C^*(S)$ is the set of elements of S which are maximal with respect to R.

IV. Clarification of the Aims of Social Policy

Social choice theory and utilitarianism seek to base social judgments on the welfares of the individual members of society; in different variants, differing assumptions are made about the properties of welfare measurements (ordinal, cardinal, interpersonally comparable cardinal, and so forth). In some articles, e.g., [II, 1979], Sen has developed a fundamental critique of this doctrine of "welfarism," as he calls it. He has argued for distinguishing aggregation of *judgments* of different people about social policy from aggregation of *interests*. For the former, only information about individual preferences is available, and the Impossibility Theorem is relevant. For aggregation of interests, on the other hand, there is additional information beyond any measure of individual welfares, in particular, measurement of inequality in objective terms. Also, the forms of consumption that give rise to pleasures might be morally relevant for given utility levels (e.g., pleasures arising from sadism and masochism). Even the Pareto principle can be questioned along these lines, and the paradox of the Pareto liberal (see Section II) shows that there can be principles which we regard as overriding the Pareto principle.

This line of argument has been developed further by Sen [1985, 1987, and 1992]. A given set of commodities may be utilized in different ways by a consumer, though the range of possible utilization modes may be restricted by an individual's personal limitations. Utility in the usual sense may be identified with happiness or fulfillment of desires. But neither the valuation to be placed on the chosen utilization nor the choice made by the individual need be related to the utility nor indeed to well-being in any sense.

Further, judgment of a given state of affairs may rationally depend, not merely on the alternative utilization chosen but on the range of alternatives available to the individual, including those not chosen. There is a preference for capability (or freedom of choice); poverty and disability are infringements on capability.

V. Measurement of Inequality and Poverty

Sen's general concerns about the meaning of social policy and individual variations in the capacity for functioning have found applications in developing appropriate measures of income inequality. His paper with Partha Dasgupta and David Starrett [III, 1973b; see also III, 1978] introduced new criteria for measures of income inequality, essentially formalizing the "transfer principle" of H. Dalton, that a transfer from a rich individual to a poor one must be regarded as a reduction in inequality. Mathematically, this leads to the implication that the function expressing inequality in terms of individual incomes must be S-concave, a weaker condition than concavity or even quasi-concavity. From this, they show that in order for one income

distribution to be better than another for *any* S-concave inequality measure, it is necessary and sufficient that the Lorenz curve for the second lie entirely below that of the first (or, equivalently, that the first be obtainable from the second by a sequence of transfers from the rich to the poor). The importance of this paper lies not merely in this characterization of the strongest statements that can be made in the absence of more definite criteria, but also in its improved tests for inequality measures.

This paper generalizes the earlier work of Atkinson (1970). However, Sen has objected that Atkinson's measures, which are additively separable in individual incomes, tend not to describe inequality very well in certain circumstances. Thus, the Gini coefficient, which is not separable in individual incomes, nevertheless satisfies a number of interesting conditions.

Closely related to measures of income inequality are measures of poverty, which have played such a large role in evaluations of public economic policies, national and international. Again, Sen [III, 1976b] has contributed importantly through an axiomatic characterization of poverty measures which take into account inequality among the poor as well as the proportion below a given level. As an approximation for a large population, he derives the poverty index,

$$P = H[I + (1 - I)G],$$

where H is the proportion in poverty, I the ratio of total income shortfalls from the poverty level to total income, and G is the Gini coefficient of income distribution among the poor.

He has pursued this analysis into the correction of national income comparisons for distributional reasons [III, 1976a]. Again an axiomatic characterization leads to the use of the Gini coefficient to modify the usual measures of *per capita* national income, specifically to measure distributionally-adjusted *per capita* national income as $e\ (1 - G)$, valid for large economies, where e is mean income and G is the Gini coefficient. This is actually a special case of a much broader set of results.

VI. Empirical Studies of Distribution and Its Consequences

The normative measures of distribution and Sen's emphasis on the functioning of individuals as the criterion for social policy have led to important studies of the interaction between income distribution and severe incapacities, including death. This is most strikingly brought out in his remarkable studies of famine [1981 and 1989]. His careful empirical analyses of four famines bring out clearly that neither variation in food supply due to natural causes nor physical obstacles to food distribution played significant roles in these famines. Rather, they were due to shifts in the real income distribution,

particularly a lowering of incomes of the very poor; it was inability to afford the purchase of food that caused hunger and famine on massive scales.

He has also studied the distribution of food within the family, particularly under conditions of stress, and has shown that there is strong evidence of discrimination against female family members in some parts of the world [VIII *passim*]. In particular, demographic analysis indicates the higher mortality of women at younger age levels.

VII. Other Areas of Study

Sen has written on many other topics than social welfare, broad as that topic is. To judge by the references, this work has had very important effects. I have followed only part of this work; in this section, I make very brief references to the questions that I have followed.

One is the meaning of rationality in individual choice. Perhaps no single paper of Sen's has been more cited than his "Rational Fools" [IV, 1977], in which he shows how limited the ordinary concept of rationality is in covering the range of motives in individual choice; in particular, the role of commitment is completely ignored. More recently, in "Maximization and the Act of Choice" [IV, 1997], he has shown that concepts of maximization are not necessarily connected with the implication of an ordering of alternatives.

Another continuing interest has been the achievement of efficiency in economic development. While the general theoretical thrust has been less novel than in his work on social welfare, his contributions have been of a very high order, especially since they frequently flew in the face of then current political dogmas accepted by many economists. This work commenced with his early book on choosing among techniques of production [1960a] and continued with many papers on the evaluation of projects culminating in the handbook on project evaluation for the United Nations International Development Organization, written jointly with Partha Dasgupta and Stephen A. Marglin [1972].

Two papers on the rate of discount appropriate to government investment [XI, 1967 and 1982] have stressed the role of externalities in the form of public concern for the future. Closely related has been his contribution to the theory of optimal savings [IX, 1961a, 1967, and 1975].

In a very different vein, Sen has addressed the idea that the value systems of India and other Asian countries are so different from that of the West that the polity and economy cannot be organized along Western lines, that democracy, for example, is inappropriate in South and East Asia. His articles [XIV, 1993a, 1993b, and 1996] have shown that a deeper knowledge of the Indian and Asiatic pasts would reveal far less uniformity and far more strands of rational analysis and democratic thinking than is asserted by self-interested participants in the debate.

VIII. Summary

One could point to still other accomplishments, especially in several different fields of philosophy, but the area where Sen's contributions have been truly unique is his extraordinary synthesis of economic and philosophical reasoning on the bases for social policy. No one has combined different approaches, formal analysis, conceptual clarification, theory of measurement and empirical work as has Sen.

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